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GYROSCOPE-FREE METHOD FOR MEASURING THE PARAMETERS OF MOVING OBJECTS

Dimitar Dichev*

Department of Machine and Precision Engineering, Faculty of Machine and Precision Engineering, Technical University of Gabrovo

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Abstract

This paper presents a novel gyroscope-free method for measuring dynamic values that define oscillations in moving objects as related to the local vertical. The method is based on appropriate algorithm which utilizes signals from linear MEMS accelerometers interconnected in a differential way. High dynamic accuracy is obtained through real time processing of measuring signals by means of algorithm that has been developed as optimization criterion for the minimum of a mean quadratic error. Said algorithm is composed according to Kalman's method and is intended to eliminate the impact of certain sources of interference each of which, individually, is of secondary importance though their cumulative action may cause a considerable distortion of the measuring signal. The method proposed eliminates the defects of conventional measuring devices used in this particular area, being based on one hand on substantially simplified mechanical module, and on the achievements in nanotechnologies, microprocessor and computer engineering on the other. In compliance with the fundamental principles of this method there has been developed a special-purpose measuring system designed to measure board and keel rolling, heeling and trim of a ship. Results from experimental investigations, carried out with specially designed test-stand equipment in shape of a hexapode and operating with six degrees of freedom, prove the efficiency of proposed measuring approach.

Keywords: gyroscope-free measuring system; Kalman's filter; Micro-electro-mechanical system (MEMS); dynamic error; board rolling, keel rolling, heeling and trim; Inertial navigation systems

1. INTRODUCTION

One of the main features distinguishing the development of measuring equipment today refers to the broader scope of application of the instruments and systems intended for measuring time-varying physical quantities. This is, to a great extent, due to the rapid development of the micro-processor and computer equipment, as well as to their successful application in improving the measuring systems used in the area of dynamic measurements [1, 2]. However, to improve the accuracy characteristics of such systems, we need to work out new and/or to improve the existing measurement methods so as to minimize or eliminate the dynamic error in the measurement result.

The development and improvement of the measuring equipment for determining the parameters of moving objects can be viewed from this perspective. For example, some of these parameters are the ones determining the position of the ship against the sea surface, such as heel, trim, etc. They are dynamic quantities. Therefore the efficiency of the ship steering depends on the accurate and the time data obtained from the ship's measuring systems in relation to the above parameters.

To ensure a specific orientation of the above moving objects and to control their motion, measuring instruments providing the required information must be mounted onboard. These instruments must include devices modeling the basic coordinate system [3, 4]. This allows us to determine the position of the moving object when rotating around its centre of mass, as well as when moving along with the latter. In addition, it enables us to keep the motion direction set. Therefore, part of the measuring instruments mounted on moving objects must possess properties that ensure continuous storage of particular directions connected with the Earth.

One of those characteristics, which is mandatory for the orientation system of most moving objects, is the local vertical. There are different methods and tools for building and keeping the local vertical in measurement mode [5, 6]. The functional elements constituting the vertical play a secondary role in organizing the overall metrological structure of the measuring instruments. Hence they are considerably important when forming the qualitative characteristics of the measuring instruments and systems in this area.

Measuring instruments built on the properties of the gyroscope are widely spread in metrology [7]. This is mainly due to the physical nature of the gyroscope that ensures its stability in relation to the inertial actions caused by the motion of the object. Therefore the dynamic accuracy of the measuring instruments built on this method is guaranteed by stabilizing the vertical in the inertial space. Under the conditions of dynamic actions these instruments provide relatively high accuracy, which reaches dynamic error values up to several tenths of a degree for the best

* *Тел.: 066827312; e-mail: dichevd@abv.bg*

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samples [5]. On the other hand, the measuring instruments built on the basis of gyro-verticals are distinguished for a number of disadvantages such as a sophisticated design, less reliability under extreme conditions, requirement of special systems ensuring the gyro-vertical operation; large sizes, high prices, etc. [6], thus limiting, to a great extent, their application.

Current achievements in science and technology provide good perspectives for developing measuring instruments of new generation, that possess better qualitative features and metrological characteristics. The actual development and improvement of such measuring instruments is first and foremost based on the fast advancement of nanotechnologies [8], microprocessor and computer equipment.

Therefore today new measuring systems whose improved accuracy parameters in dynamic measurement mode are formed on the basis of new measurement concepts and signal processing algorithms can be implemented. On the basis of their improved characteristics, they can more effectively replace the current measuring instruments based on the gyroscopic principle of vertical stabilization.

2. A concept of modeling measuring instruments

The proposed measurement concept can be successfully used for modeling instruments that measure the angular position of moving objects in relation to the basic coordinate system, as well as their dynamic fluctuations around their instantaneous axis of rotation. It is designed for developing new measuring instruments in this area since it is based on a different approach targeting the elimination of the dynamic error caused by the deviation of the inertial components that model the vertical from the inertial space in real time rather than their stabilization. This modeling concept can overcome the disadvantages of the existing measuring instruments as it is based, on one hand, on a very simplified mechanical module, and on the other hand, on the possibilities of modern measuring equipment in the area of dynamic measurements. In addition, it is based on successfully integrated processing algorithms [8, 9, 10] intended for eliminating the dynamic error [9, 11].

The block diagram illustrating the operating principle of the measuring systems developed according to the concept of the present approach is shown in fig. 1 [1, 9]. In general it consists of a main measurement channel, an additional channel, interfaces to connect with a computer and programme modules for processing and presenting measurement information.

The main channel is used for measuring the current values of the angles determining the position of the moving object in the basic coordinate system. It is based on the gyro-free principle of vertical modeling and a very simplified design, which results in considerably reducing the magnitude of the instrumental errors.

However, this way of modeling the vertical leads to its instability in the inertial space. Under inertial actions caused by the motion and fluctuations of the object, this instability determines the deviation of the elements modeling the vertical from the actual direction of the vertical. All this results in a dynamic error which in some cases can reach inadmissible high values [12]. Therefore the proposed concept envisages a procedure eliminating the current values of the dynamic error from the measurement result.

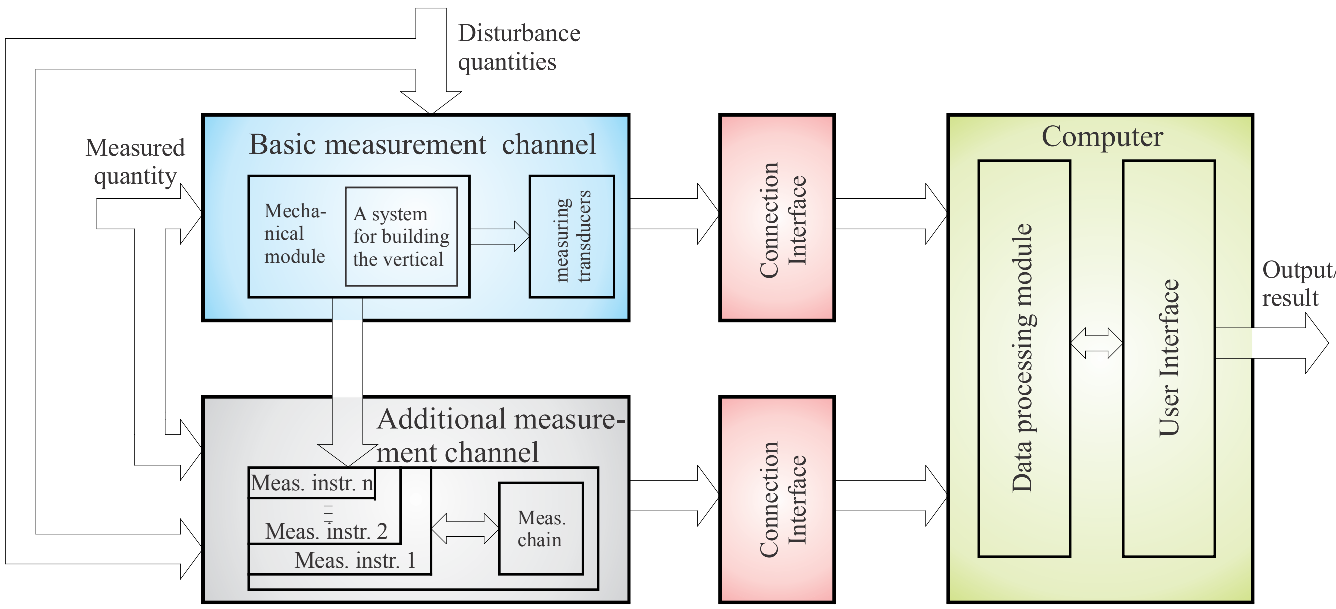

Fig.1. *Block diagram illustrating the principle of modeling of measuring systems*

The procedure related to the obtaining of the measurement information required for determining the current values of the dynamic error is implemented in the additional channel (fig.1). The latter operates in parallel with the main channel, which provides an opportunity to eliminate the dynamic error from the measurement result in real time. The structure of the additional channel and the type of devices constituting it are specified on the basis of the selected model for determining the current values of the dynamic error and the algorithm for correcting the signal from the main measurement channel.

3. A measuring system for determining the heel and trim of a moving object

To illustrate the characteristics of the proposed concept, a specific measuring system developed in compliance with the principles of this concept is presented. The system is designed for measuring the roll, pitch, heel and trim of a ship. Its block diagram is shown in fig. 2 [1, 9].

The basic concept of the present measuring system is focused on the simplified design of the vertical in the form of a physical pendulum (fig.3). This comparatively simple design of the mechanical module, consisting of a small number of elements, results in limiting the magnitude of the instrumental error. The body 1 of the mechanical module is fixed to the ship. By means of an appropriate suspension system 2, a system for modeling the vertical 3 is mounted on the body. The latter consists of an outer frame 1 and inner frame 2, connected in series by cylindrical joints (fig.4). A physical pendulum of two degrees of freedom 3 is attached to the inner frame (fig.4). The two frames have interperpendicular axes of rotation, which intersect at one point. The measurement information about the heel and trim angles is obtained from identical absolute encoders 4 and 5, mounted on the respective measurement axis.

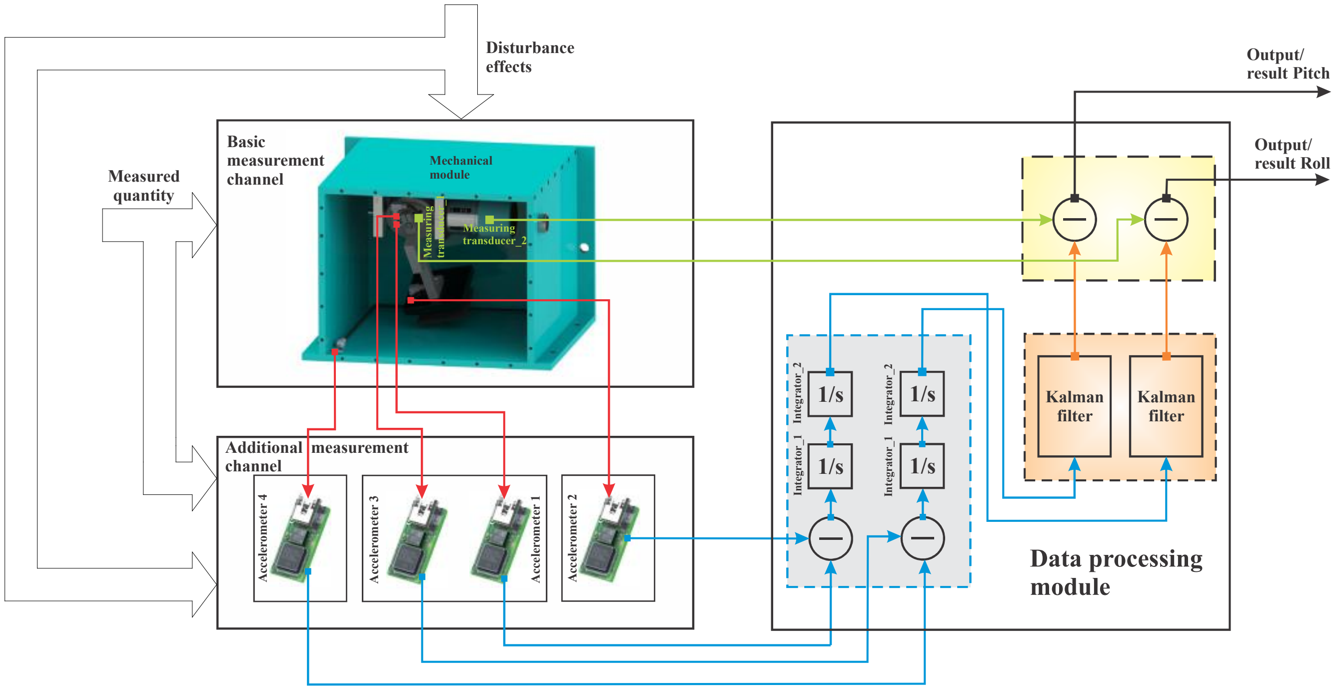

Fig. 2. Block diagram of the measuring system

The absolute encoders 4 and 5 used in the model are intended for transforming the angular displacements in coded electrical signals corresponding to the absolute position of the shaft. The application of differential parallel scanning of each division of the rotating scale in Gray code eliminates the errors due to interferences and provides a wide operating temperature range. The absolute encoders are distinguished for their high accuracy, high noise immunity, fast response, wide range of supply voltage and small size.

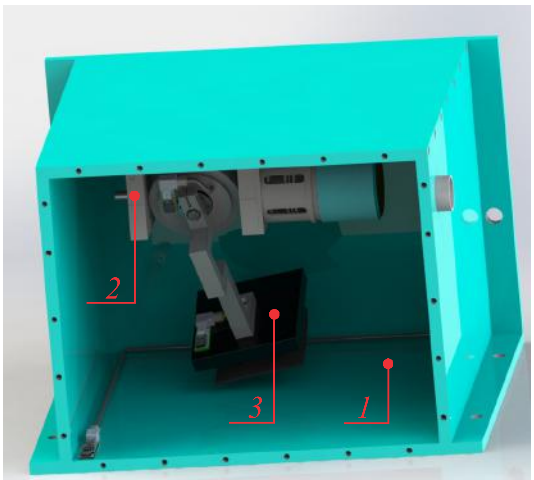

Fig. 3. Design model of the main measurement channel

The measurement accuracy in dynamic mode is ensured by an additional channel for determining the dynamic error. It consists of two pairs of identical MEMS accelerometers used for measuring the linear acceleration. The accelerometers are mounted respectively on the body of the mechanical module, on the first cylindrical joint (two accelerometers), and on the physical pendulum (fig.2). The first two accelerometers are mounted in such a way that their measurement axes are sensitive to the accelerations generated by the roll whereas the measurement axes of the other two accelerometers are sensitive to the accelerations generated by the pitch. This scheme of mounting of the MEMS sensors ensures the sensitivity of the first accelerometer of each sensor pair to all accelerations generated by the roll and the pitch. Every second accelerometer is sensitive not only to the accelerations of the first sensor but also to those generated by the pendulum motion in relation to its degree of freedom. This makes possible the development of a procedure involving the subtraction of the signals from the first and second accelerometer of each sensor pair where the output signals are proportional to the accelerations generated by the pendulum motion in relation to its degree of freedom. By means of a data processing algorithm a double integration of the accelerations is performed, where signals defining the pendulum deviations from the vertical in relation to its two degrees of freedom are obtained.

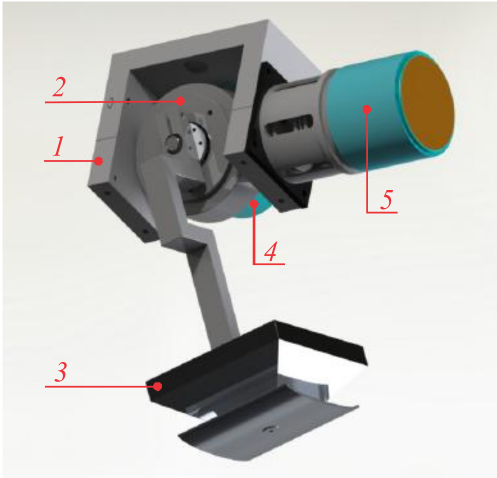

Fig. 4. *System modeling the vertical of the device*
1 – outer frame; 2 – inner frame; 3 - physical pendulum; 4 and 5 - absolute encoders of FKP 13.A type and 213-bit resolution

Usually the measuring instruments operating in similar mode are subjected to other interference actions whose sources are external or internal additional secondary processes of unpredictable behaviour. Hence it is necessary to introduce an additional procedure for protecting the measuring instrument against the presence of these interference sources. Taking into account the characteristics of the quantities constituting the measurement environment where the measuring instruments under consideration operate, it can be concluded that the best form of eliminating the influence of the interference sources is the Kalman filter. The characteristics of this algorithm fit very well into the solution of a number of problems emerging in the process of optimization of the accuracy characteristics of the measuring instruments defining the parameters of the above listed moving objects [8, 10, 13, 14, 17]. Therefore a module for signal processing by means of the Kalman algorithm is included in each measurement channel of the block diagram (fig.2).

4. A mathematical model of the measuring system

The dynamic characteristics of the instruments under consideration refer to their metrological characteristics as they affect the formation of the error obtained as a result of the measurement. The mathematical model presents the main ratios of the measuring system, the measured and the interfering quantities, in a form which is suitable for analytical study of the dynamic characteristics [9]. The model is developed on the basis of the block and the operating diagrams described above. The differential equations related to the motion of the instruments' inertial components present the most complete description of the dynamic characteristics of the instruments. The equations are worked out for operating conditions close to the real ones. The latter are mainly defined by the ship's motion in real wind-generated rough seas. They represent a sophisticated dynamic process which can be considered as a set of deviations according to each degree of freedom.

In order to make the mathematical operations easier, only one of the two measuring channels will be viewed - that related to the trim. In addition, the mathematical model of the accelerometer mounted on the cylindrical joint can be easily deduced from the equations of the second accelerometer positioned on the instrument pendulum, due to which it will not be taken into account when working out the differential equations.

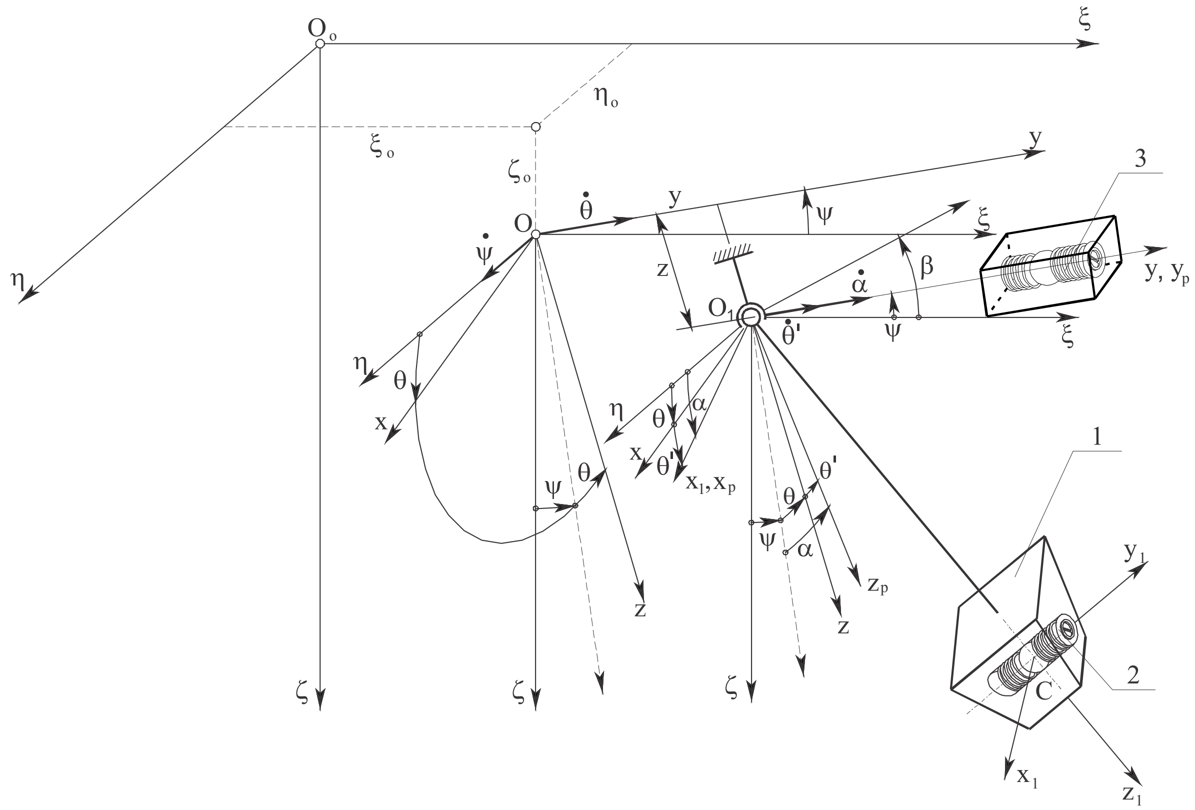

Fig. 5. *Diagram of the dynamic system moving object - measuring instrument*

1 – physical pendulum; 2 – first accelerometer; 3 – second accelerometer; O_o,η,ξ,ζ *- supporting coordinate system;* O,x,y,z *– coordinate system connected to the ship;* C,x_1y_1,z_1 *– coordinate system connected to the physical pendulum;* O,x_p,y_p,z_p *– intermediate coordinate system;* η_o, ξ_o, ζ_o *– coordinates of the position of the ship's centre of gravity in relation to the supporting coordinate system*

The differential equations are worked out on the basis of a dynamic system shown in Fig. 5. The motions of a ship are defined as angular and linear fluctuations of a rigid body around or along with its centre of gravity. The moving object (the ship), to which the coordinate system O,x,y,z is connected, changes its position randomly in relation to the supporting system O_o,ξ,η,ζ. The measuring instrument is mounted on the ship and its sensor (the physical pendulum) is connected to the coordinate system $Cx_1y_1z_1$. The suspension point O_1 of the instrument's sensor coincides with the diametral plane of the ship and its position with regard to the centre of gravity of the moving object O is defined by the z and y coordinates. The position of the moving object in relation to the supporting system O_o,ξ,η,ζ is set by the three coordinates of its centre of gravity O - ξ_o, η_o, ζ_o and the matrix $A=\left\|a_{ij}\right\|$ $(i,j=1,2,3)$ of the given angle cosines of the trim ψ and the heel θ, defining the angular displacement between the axes of the systems $O_o\xi\eta\zeta$ and $Oxyz$. The mechanical system consists of two bodies - a physical pendulum which is free to rotate with regard to the coordinate axes O_1x_1 and O_1y_1, and an accelerometer mounted in the centre of gravity C of the physical pendulum. The accelerometer's inertial body of a mass m_2 stays at an equilibrium position in relation to the y_1 coordinate by means of two horizontal springs of an elastic constant c.

Therefore, the mechanical system has three degrees of freedom and the generalized coordinates are, α, β and y_1, respectively. The α and β coordinates define the angular displacement of the physical pendulum from the vertical in relation to O_1y_1 and O_1x_1 axes, respectively, whereas y_1 determines the relative motion of the inertial mass m_2. By means of the β coordinate the inertial component of the dynamic error for the measuring channel under consideration is defined. The latter determines the trim values of a ship.

The Lagrangian method is used upon working out the differential equations. The kinetic energy of the system is the sum of the kinetic energies of two bodies. The first one is a physical pendulum of two degrees of freedom and the second is the moving mass of an accelerometer of one degree of freedom.

$$E_k=E_{k1}+E_{k2}. \tag{1}$$

The kinetic energy of the first body is defined according to König's theorem related to rigid bodies, i.e.:

$$E_{k1}=\frac{1}{2}.m_1.V_C^2+\frac{1}{2}.J_{C\omega}.\omega^2, \tag{2}$$

where m_1 – the mass of the first body; $V_C=\sqrt{[\dot{\eta}_C(t)]^2+[\dot{\xi}_C(t)]^2+[\dot{\zeta}_C(t)]^2}$ - the absolute velocity of the point C; $J_{C\omega}$ - the body's moment of inertia in relation to the current axis C_ω through the body centre of mass; ω - the absolute velocity of the body.

Since the $C,x_1y_1z_1$ coordinate system is constantly connected to the physical pendulum, its inertial characteristics remain constant in time. In this case, the sensor's mass moments of inertia with regard to the coordinate axes of $C,x_1y_1z_1$ remain constant. Then, for the second addend in (2), which defines the rotary motion of the sensor, the following is obtained:

$$\begin{aligned}T_r=\frac{1}{2}.J_{C\omega}.\omega^2=\frac{1}{2}.\Big(&J_{x_1}.\omega_{x_1}^2+J_{y_1}.\omega_{y_1}^2+J_{z_1}.\omega_{z_1}^2-\\&-2.J_{x_1y_1}.\omega_{x_1}.\omega_{y_1}-2.J_{x_1z_1}.\omega_{x_1}.\omega_{z_1}-2.J_{y_1z_1}.\omega_{y_1}.\omega_{z_1}\Big),\end{aligned} \tag{3}$$

where J_{x_1},J_{y_1},J_{z_1} are the sensor's mass moments of inertia in relation to the respective axes of the $C,x_1y_1z_1$ system; $J_{x_1y_1},J_{x_1z_1},J_{y_1z_1}$ are the centrifugal mass moments of inertia of the body with regard to the respective axes of the $C,x_1y_1z_1$ system.

The axes of $Cx_1y_1z_1$ are assumed to be the principal axes of inertia of the body, i.e. $J_{x_1y_1}=J_{x_1z_1}=J_{y_1z_1}=0$, by means of which the above expression is simplified to

$$T_r=\frac{1}{2}.\Big(J_{x_1}.\omega_{x_1}^2+J_{y_1}.\omega_{y_1}^2+J_{z_1}.\omega_{z_1}^2\Big). \tag{4}$$

The projections of the angular velocity ω on the axes of the $Cx_1y_1z_1$ system can be derived from fig. 3, and their final expression is:

$$\begin{aligned}&\omega_{x_1}=\dot{\beta}+\dot{\psi}.cos\,\alpha;\\&\omega_{y_1}=\dot{\theta}.cos(\beta-\psi)+\dot{\alpha}.cos(\beta-\psi)+\dot{\psi}.sin\,\alpha.sin(\beta-\psi);\\&\omega_{z_1}=\dot{\theta}.sin(\beta-\psi)+\dot{\alpha}.sin(\beta-\psi)-\dot{\psi}.sin\,\alpha.cos(\beta-\psi).\end{aligned} \tag{5}$$

The kinetic energy of the second body is equal to the sum of the kinetic energies of the two components of the absolute motion, i.e.:

$$E_{k2}=\frac{1}{2}.m_2.V_m^2+\frac{1}{2}.m_2.\dot{y}_1^2, \tag{6}$$ (1)

where m_2 – the mass of the inertial component of the accelerometer; $V_m=\sqrt{[\dot{\eta}_m(t)]^2+[\dot{\xi}_m(t)]^2+[\dot{\zeta}_m(t)]^2}$ - (2) the absolute velocity of the mass m_2; $\dot{y}_1$ - the relative velocity of the mass m_2.

After doing all necessary mathematical operations for the final definition of (2) and (6), and after substituting in (1), the following final formula for the kinetic energy of the dynamic system under study is obtained:

$$
\begin{aligned}
E_k = {} & q_1.\dot{\eta}_o^2 + q_2.\dot{\xi}_o^2 + q_3.\dot{\zeta}_o^2 + q_4.\dot{\theta}^2 + q_5.\dot{\psi}^2 + q_6.\dot{\eta}_o.\dot{\theta} + q_7.\dot{\xi}_o\dot{\theta} + q_8.\dot{\zeta}_o.\dot{\theta} + q_9.\dot{\eta}_o.\dot{\psi} + \\
& + q_{10}.\dot{\xi}_o.\dot{\psi} + q_{11}.\dot{\zeta}_o.\dot{\psi} + q_{12}.\dot{\theta}.\dot{\psi} + q_{13}.\dot{\beta}^2 + q_{14}.\dot{\alpha}^2 + q_{15}.\dot{\eta}_o.\dot{\beta} + q_{16}.\dot{\xi}_o.\dot{\beta} + q_{17}.\dot{\zeta}_o.\dot{\beta} + \\
& + q_{18}.\dot{\eta}_o.\dot{\alpha} + q_{19}.\dot{\xi}_o.\dot{\alpha} + q_{20}.\dot{\zeta}_o.\dot{\alpha} + q_{21}.\dot{\theta}.\dot{\beta} + q_{22}.\dot{\theta}.\dot{\alpha} + q_{23}.\dot{\psi}.\dot{\beta} + q_{24}.\dot{\psi}.\dot{\alpha} + q_{25}.\dot{\alpha}.\dot{\beta} + \\
& + q_{26}.\dot{y}_1^2 + q_{27}.\dot{y}_1.\dot{\beta} + q_{28}.y_1.\dot{y}_1.\dot{\beta} + q_{29}.y_1.\dot{y}_1.\dot{\alpha} + q_{30}.\dot{y}_1.\dot{\alpha} + q_{31}.\dot{y}_1.\dot{\zeta}_o + q_{32}.\dot{y}_1.\dot{\xi}_o + \\
& + q_{33}.\dot{y}_1.\dot{\theta} + q_{34}.\dot{y}_1.\dot{\psi} + q_{35}.y_1^2.\dot{\beta}^2 + q_{36}.y_1^2.\dot{\alpha}^2 + q_{37}.y_1^2.\dot{\beta}.\dot{\alpha} + q_{38}.y_1.\dot{\beta}^2 + q_{39}.y_1.\dot{\beta}.\dot{\alpha} + \\
& + q_{40}.y_1.\dot{\zeta}_o.\dot{\beta} + q_{41}.y_1.\dot{\zeta}_o.\dot{\alpha} + q_{42}.y_1.\dot{\xi}_o.\dot{\beta} + q_{43}.y_1.\dot{\theta}.\dot{\beta} + q_{44}.y_1.\dot{\psi}.\dot{\beta} + q_{45}.y_1.\dot{\psi}.\dot{\alpha} + \\
& + q_{46}.y_1.\dot{\theta}.\dot{\alpha} + q_{47}.y_1.\dot{\alpha}^2 ,
\end{aligned} \tag{7}
$$

where q_1, q_2, …, q_{47} are the coefficients depending on the design parameters of the instrument's inertial components, on the angular quantities representing time functions and determining the position of the ship in relation to the water surface and the position of the pendulum with regard to the ideal vertical, as well as on the geometric parameters defining the position of the measuring instrument in relation to the ship's centre of gravity.

The number of Lagrange's equations used for writing the differential equations are three as well, since the generalized co-ordinates α, β and y_1, in relation to which some solutions of the model are sought, are three. After defining the generalized forces and substituting them in Lagrange's equations along with the derivatives of (7) in relation to the generalized coordinates and time, we obtain the differential equations of the dynamic system under study, whose matrix form could be written as follows:

(8)

$$
\begin{Vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{Vmatrix} . \begin{Vmatrix} \ddot{\alpha} \\ \ddot{\beta} \\ \ddot{y}_1 \end{Vmatrix} + \begin{Vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{Vmatrix} . \begin{Vmatrix} \dot{\alpha}^2 \\ \dot{\beta}^2 \\ \dot{y}_1^2 \end{Vmatrix} + \begin{Vmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{Vmatrix} . \begin{Vmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{y}_1 \end{Vmatrix} +
$$

$$
+ \begin{Vmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{Vmatrix} . \begin{Vmatrix} \alpha \\ \beta \\ y_1 \end{Vmatrix} + \begin{Vmatrix} d_1 \\ d_2 \\ d_3 \end{Vmatrix} .\dot{\alpha}.\dot{\beta} + \begin{Vmatrix} h_1 \\ h_2 \\ h_3 \end{Vmatrix} .\dot{y}_1.\dot{\alpha} + \begin{Vmatrix} g_1 \\ g_2 \\ g_3 \end{Vmatrix} .\dot{y}_1.\dot{\beta} +
$$

$$
+ \begin{Vmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{Vmatrix} . \begin{Vmatrix} y_1.\ddot{\alpha} \\ y_1.\ddot{\beta} \\ y_1.\ddot{y}_1 \end{Vmatrix} + \begin{Vmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{Vmatrix} . \begin{Vmatrix} y_1.\dot{\alpha}^2 \\ y_1.\dot{\beta}^2 \\ y_1.\dot{y}_1^2 \end{Vmatrix} + \begin{Vmatrix} u_1 \\ u_2 \\ u_3 \end{Vmatrix} .y_1.\dot{\alpha}.\dot{\beta} =
$$

$$
= \begin{Vmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{Vmatrix} . \begin{Vmatrix} \ddot{\eta}_o \\ \ddot{\xi}_o \\ \ddot{\zeta}_o \end{Vmatrix} + \begin{Vmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \\ e_{31} & e_{32} \end{Vmatrix} . \begin{Vmatrix} \ddot{\theta} \\ \ddot{\psi} \end{Vmatrix} + \begin{Vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \\ f_{31} & f_{32} \end{Vmatrix} . \begin{Vmatrix} \dot{\theta}^2 \\ \dot{\psi}^2 \end{Vmatrix} +
$$

$$
+ \begin{Vmatrix} n_1 \\ n_2 \\ n_3 \end{Vmatrix} .\dot{\theta}.\dot{\psi} + \begin{Vmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \\ p_{31} & p_{32} \end{Vmatrix} . \begin{Vmatrix} \dot{\theta}.\dot{\alpha} \\ \dot{\theta}.\dot{\beta} \end{Vmatrix} + \begin{Vmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \\ r_{31} & r_{32} \end{Vmatrix} . \begin{Vmatrix} \dot{\psi}.\dot{\alpha} \\ \dot{\psi}.\dot{\beta} \end{Vmatrix} ,
$$

where the elements of the separate matrixes are functions of the parameters defining the geometric and the mass inertial characteristics of the measuring instrument, as well as the quantities determining the position of the ship and the instrument's sensors with regard to their equilibrium position.

The differential equations are formed under the influence of a great number of parameters which can be divided in the following groups, depending on the defining factors: parameters defining the characteristics of the sea roughness; parameters defining the shape, the dimensions, the geometric and the mass characteristics of the ship; parameters defining the position of the vessel in relation to the wave direction and kinematics parameters defining its motion; hydrodynamic parameters defining the interaction between the water and the ship; geometric and mass parameters defining the design of the instrument; as well as parameters defining its position with regard to the ship's centre of gravity.

The mathematical model determined by equations (8) describes best the properties and characteristics of the measuring system because it expresses the interrelation between the system's readings and the values of the measured quantity, the design parameters and the influencing quantities. This model allows us to predict the measurement results when the system operates under different running conditions, as well as to optimize its design and parameters, thus subjecting their selection to the conditions that best realize a minimum error measurement.

Equations (8) are reduced to simplified form by linearizing the quantities defining the movement of the system's points and by including only small first-order quantities. As a result, the following system of differential equations is obtained:

$$
\left|
\begin{aligned}
& \left(J_{y_1} + m_1.l^2\right).\ddot{\alpha} + k_\alpha.\dot{\alpha} + m_1.g.l.\alpha = \\
& = m_1.l.\ddot{\eta}_o - \left(J_{y_1} + m_1.l.z\right).\ddot{\theta}\,; \\
& \left(J_{x_1} + m_1.l^2\right).\ddot{\beta} + k_\beta.\dot{\beta} + m_1.g.l.\beta = \\
& = -m_1.l.\ddot{\xi}_o - \left(J_{x_1} + m_1.z.l\right).\ddot{\psi}\,; \\
& m_2.\ddot{y}_1 + k_{y_1}.\dot{y}_1 + c.y_1 = \\
& = -\frac{1}{2}m_2.\left(l.\ddot{\beta} + \ddot{\xi}_0 + z.\ddot{\psi}\right),
\end{aligned}
\right. \tag{9}
$$

where k_α, k_β and k_{y1} are damping coefficients according to the three generalized coordinates α, β and y_1; l – the length of the physical pendulum.

The third equation in (9) defines the link between the readings of the accelerometer mounted on the physical pendulum and the quantities entering the input of the instrument. A differential equation representing the motion of the sensor of the second accelerometer can be easily worked out from this equation, taking into account the identical design characteristics of the two sensors and the fact that this accelerometer is not sensitive to the motion of the physical pendulum. Then the differential equation will be:

$$m_2.\ddot{y}_p + k_{y_p}.\dot{y}_p + c.y_p = -\frac{1}{2}m_2.\left(\ddot{\xi}_0 + z.\ddot{\psi}\right), \tag{10}$$

where y_p is the coordinate of the motion of the sensor of the second accelerometer.

In this case the difference between the readings of the two accelerometers will be proportional to the function $\ddot{\beta}(t)$, by means of which we can easily determine the quantity $\beta(t)$ defining the dynamic error.

Actually, additional interferences are superimposed on the signal. They can be easily identified if equations (8) are set in a form where first- and second-order quantities are used, i.e.:

$$\left|\begin{aligned}
&\left(J_{y_1} + m_1.l^2\right).\ddot{\alpha} + b_1.\dot{\alpha} + m_1.g.l.\alpha = \\
&= m_1.l.\left(\ddot{\eta}_0 + \alpha.\ddot{\zeta}_0\right) - \left(J_{y_1} + m_1.l.z\right).\ddot{\theta} + \\
&+ m_1.l.y.\left(\theta.\ddot{\psi} - \alpha.\ddot{\psi} - 2.\dot{\alpha}.\dot{\psi} + 2.\dot{\theta}.\dot{\psi} + \psi.\ddot{\theta}\right) \\
&\left(J_{x_1} + m_1.l^2\right).\ddot{\beta} + 2.m_2.l.\beta.\ddot{y}_1 + b_2.\dot{\beta} + \\
&+ m_1.g.l.\beta = m_1.l.\left(\beta.\ddot{\zeta}_0 - \ddot{\xi}_0\right) + \\
&+ m_2.\left(\ddot{\zeta}_0 - y.\ddot{\psi}\right).y_1 - \left(J_{x_1} + m_1.z.l\right).\ddot{\psi} + \\
&+ m_1.y.l.\left(\psi.\ddot{\psi} - \beta.\ddot{\psi} + \dot{\psi}^2\right) \\
&m_2.\ddot{y}_1 + k_{y_1}.\dot{y}_1 + c.y_1 = \frac{1}{2}.m_2.\left(\ddot{\zeta}_o.\beta - l.\ddot{\beta} - y.\ddot{\psi}.\beta\right) + \\
&+ \frac{1}{2}.m_2.\left[y.\left(\ddot{\psi}.\psi + \dot{\psi}^2\right) - z.\ddot{\psi} - \ddot{\xi}_o\right].
\end{aligned}\right. \tag{11}$$

It can be seen in the third equation in (11), that the functions $\ddot{\zeta}_o.\beta$ and $y.\ddot{\psi}.\beta$ appear on the right hand side. They are superimposed on the signal $\ddot{\beta}(t)$. Although the values of these functions are formed as small quantities of second order in relation to $\ddot{\beta}(t)$, they introduce an additional error when determining the quantity $\beta(t)$. Therefore, they have to be eliminated from the signal before the final formation of the function $\beta(t)$. In this case, the use of Kalman filter is very suitable. Its position in the measuring procedure is given in the operating diagram shown in Fig. 2.

5. EXPERIMENTS

To carry out the experiments, the required stand equipment has been developed [1]. It is a hexapod of six degrees of freedom, which makes possible the reproduction of the fluctuations of the ship in a form close to the real operating conditions [15, 16, 18]. To ensure accuracy, the equipment is calibrated and metrological traceability of its unit to the length standard is provided. The equipment is shown in fig. 6.

Fig. 6. *Stand equipment*
1 – operating platform; 2 - actuators; 3 – immobile base; 4 – connection and control interface; 5 – computer

Figure 7 illustrates the results from the investigation of the dynamic accuracy of the measuring system in its two operating modes [1]. Within the first operating mode a Kalman filter is not used. Unlike it within the second operating mode a module processing the measurement signals in compliance with the Kalman algorithm is used. The curves in fig. 7 show the dynamic errors for both operating modes in a graphic format. The errors, respectively without $\varepsilon_{de}(t)$ and with $\varepsilon_{de}^{kf}(t)$ a module using a Kalman filter, are determined by the expressions

$$\varepsilon_{de}(t) = \psi_{mr}(t) - \psi(t); \tag{12}$$

$$\varepsilon_{de}^{kf}(t) = \psi_{mr}^{kf}(t) - \psi(t), \tag{13}$$

where $\psi_{mr}(t)$ and $\psi_{mr}^{kf}(t)$ are the functions obtained as a result of the measurement of the platform motion, respectively without and with a Kalman filter; $\psi(t)$ – the function defining the motion of the operating platform in relation to the trim coordinate.

Errors $\varepsilon_{de}(t)$ and $\varepsilon_{de}^{kf}(t)$ are time functions since according to (12) and (13) both the measurement result and the referent quantity are dynamically changing processes. Function $\psi(t)$ defining the referent motion of the operating platform along the trim coordinate is obtained as a result of the constitutive motions of the platform along its six degrees of freedom. The specific motion of the operating platform, upon measuring of which errors $\varepsilon_{de}(t)$ and $\varepsilon_{de}^{kf}(t)$ in fig .7 are determined, is shown in fig.8.

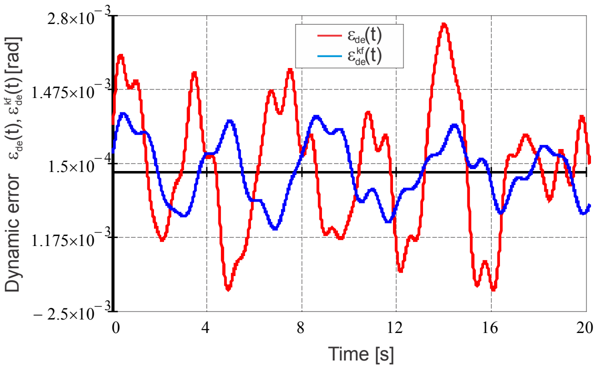

Fig. 7. *Results from the investigation of the dynamic accuracy*

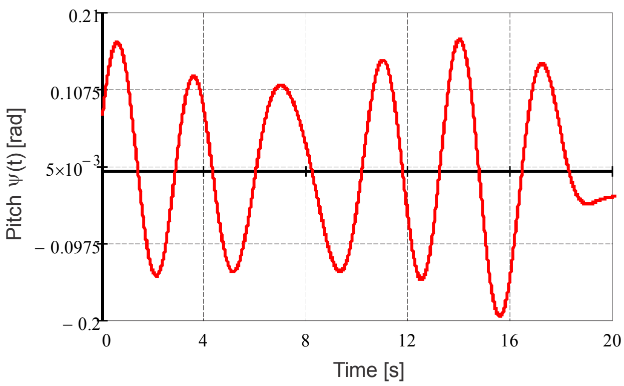

Fig. 8. *Motion of the operating platform along the angular coordinate* ψ

As it can be seen in fig.7 (the red curve), the measuring system is stable enough in relation to its dynamic accuracy even without using the Kalman algorithm. In this case the maximum values of the dynamic error are within a range of *0,15° - 0,16°*. The experiments show that even under the most severe conditions caused by the fluctuations of the ship the error does not exceed *0,2° - 0,3°*.

The Kalman algorithm considerably improves the dynamic accuracy of the measuring system, which is illustrated in fig. 7 (the blue curve). It can be seen that the maximum values of the dynamic error $\varepsilon_{de}^{kf}(t)$ vary within a range of *0,05° - 0,07°*.

6. CONCLUSIONS

The proposed measurement concept is designed for developing gyro-free measuring systems that determine the parameters of moving objects. This modeling approach overcomes the disadvantages of the existing measuring instruments since it is based, on one hand, on a very simplified mechanical module, and on the other hand, on the advanced achievements in the area of nanotechnologies, microprocessor and computer equipment.

The high dynamic accuracy of the proposed measuring system is ensured by an additional measurement channel operating in parallel with the main channel. The metrological procedures in the additional channel are based on an appropriate correction algorithm using signals from linear MEMS accelerometers.

The experimental results confirm the effectiveness of the proposed measurement concept in relation to the dynamic accuracy of systems measuring moving objects. As a result of the operation of the additional channel and the Kalman algorithm the accuracy characteristics of the measuring system under conditions of dynamic influences are improved to a great extent. This can be implemented without using expensive elements and stabilization systems.

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