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# STIFFNESS FACTOR FORMATION UNDER STRUCTURAL CIRCUIT SYNTHESIS OF INDUSTRIAL ROBOTS WIRE-FRAME ARRANGEMENTS

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**Abstract**

*It is suggested that the best solutions during the structural circuit synthesis of wireframe arrangements for industrial robots with and without the mechanisms of parallel structure are chosen on account of the integrated stiffness index based on the relative stiffness factor (where the round rod of the same cross section and length is taken as a rating criterion) and stiffness instability factor within a manipulated object travelling area.*

**Keywords:** stiffness factor, wire-frame arrangement, industrial robot, structural circuit synthesis, parallel structure mechanism

## 1. INTRODUCTION

Robots with different arrangements and applications are designed based on a variety of criteria so that their structure and functionality can be assessed and improved [7, 13 - 16, 20]. The technical systems theory [6, 12] divides all criteria into four groups: functional, technical, economic, ergonomic ones. These are also applicable to industrial robots in the works [15, 16], where the accuracy rating is included into functional criteria, but the stiffness index, which affects accuracy, is not. Maneuverability, load-carrying capacity, positioning accuracy, high speed response and reliability have been selected among the range of indices for industrial robots and mobile platforms in parallel structure mechanisms (PSM) in the works [20 - 22]. These properties are proposed to be evaluated with various quantitative indices, in which the arrangement and carrier system stiffness of an industrial robot are not taken into account [14, 20].

Only in the works [8, 10, 16], the authors attribute the reliable performance of industrial robots to stiffness and the dynamics of their carrier systems during the design parameters optimization.

A number of works draw attention to the choice of industrial robots arrangements and robotic systems based on a modular principle [1, 8, 10, 15, 16, 19] with system morphological approach applied for synthesis [9, 10 - 12, 16, 22], where it is advisable to use the stiffness and dynamic stability of the wire-frame structures as one possible criterion for selection of the best solutions [3, 5, 8, 18].

## 2. The essence of the problem and its solution

When the load F is applied to the rod AB as the weight of a manipulated object, the latter is stretched (compressed) by a BB' value (Fig 1 a.) [17]:

$$\Delta_1 = \frac{FL}{ES},$$

where $l$ is the rod’s length; $E$ is elasticity modulus; $S$ is the rod area equal to $\approx 0.8d^2$; $d$ is the rod’s diameter.

If the rod’s length is doubled ($L = 2l$), it will stretch by a BB’’ value under the load F (Figure 1, b.):

$$\Delta_2 = \frac{FL}{ES} = \frac{2Fl}{ES}$$

In the second case $\Delta_2 = 2\Delta_1$, and the comparison with the first version—the measure of stiffness can be represented as

$$K^P_{Ж2} = \frac{\Delta_1}{\Delta_2} = \frac{1}{2}$$

In general, the stiffness ratio of i[th] version compared will be:

$$K^P_{Ж2} = \frac{\Delta_1}{\Delta_i}, \tag{1}$$

where $\Delta_1$ is the rating index (model) deformation; $\Delta_i$ is the deformation of B butt end of i[th] version of industrial robot element compared (carrier system or a manipulator mechanical arm) under tension or compression.

When the load F is applied to the cantilevered sealed in rod (Figure 1, c), adopted as the rating index for bending stiffness, the B butt end of the rod is moved by the BB’ value [7, 17]:

$$\Delta_{H1} = \frac{Fl^3}{3EI},$$

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where I is the cross section moment of inertia equal to $\approx 0,05d^4$.

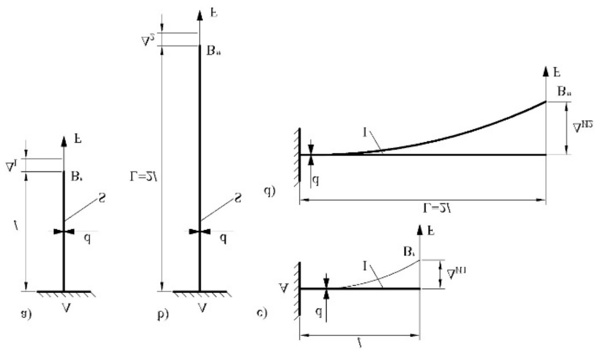

*Fig. 1. Rating indices—round rods being stretched (compressed) (a) and bent (c) rods, and the versions (b, d) being compared based on the stiffness factor.*

In the second case (Figure 1, d) the cantilever beam’s end is moved by the value of BB”:

$$\Delta_{И2} = \frac{FL^3}{3EI} = \frac{F(2l)^3}{3EI} = \frac{8Fl^3}{3EI}.$$

As per the adopted approach, stiffness ratio with regard to the second version bent will be as follows as compared with the model

$$K^{И}_{Ж2} = \frac{\Delta_{И1}}{\Delta_{И2}} = \frac{1}{8},$$

and in general

$$K^{И}_{Жi} = \frac{\Delta_{И1}}{\Delta_{Иi}}, \qquad (2)$$

where $\Delta_{Иi}$ is the deformation of cantilever butt end B of i[th] version in the spot where the load F is applied.

For frames shaped as curved rods under assumptions made (tight coupling of rods excluding gaps in joints and hinges, anchorage) and based on the superposition principle [3, 17, 18], the total deformation in the point where the load F is applied will approximately be:

$$\Delta_{\Sigma i} = \Delta_i + \Delta_{Пi} + \Delta_{Иi}, \qquad (3)$$

where $\Delta_i$ is the deformation of the butt end C of i[th] version compared on stretching or compressing of AB rod by the force F; $\Delta_{Пi}$ is the deformation of butt end C of i[th] version compared when the non-deformable rod BC is turned by the momentum from the force F moving from point C to point B; $\Delta_{Иi}$ is the deformation of butt end C of the version compared when bent.

The deformation $\Delta_{Пi}$ is determined by the method of direct integration based on main differential equation [17] $\frac{d^2\omega}{dx^2} = \frac{M(x)}{EI(x)}$, in which the value of the bending (torque) momentum is calculated for the non-deformed beams.

Solution of the main differential equation gives the final deflection and rotational angle solutions:

$$\omega(x) = -\frac{Fl^3}{6EI}[2 - 3\frac{x}{l} + (\frac{x}{l})^2]; \qquad (4)$$

$$v(x) = \frac{Fl^2}{2EI}[1 - (\frac{x}{l})^2]; \qquad (5)$$

Elastic line of the beam (4) is a third degree parabolic curve.

At x = 0 the rotation angle on the free end of the beam is $v_1 = \frac{Fl^2}{2EI}$ for versions (fig. 2, a, b) and $v_2 = \frac{FL^2}{2EI} = \frac{F(2l)^2}{2EI} = \frac{4Fl^2}{2EI} = \frac{2Fl^2}{EI}$ (fig. 2, c, d).

The rotation-induced deformation will thus be

$$\Delta_{Пi} = v_i \cdot l_i, \qquad (6)$$

hence $\Delta_{П1} = v_1 \cdot l = \frac{Fl^3}{2EI}$ (fig.2, a);

$\Delta_{П2} = 2v_1 \cdot l = \frac{Fl^3}{EI}$ (fig.2, b); $\Delta_{П3} = v_2 \cdot l = \frac{2Fl^3}{EI}$ (fig.2, c); $\Delta_{П3} = 2v_2 \cdot l = \frac{4Fl^3}{EI}$ (fig.2, d);

In general, the stiffness ratio of i[th] version compared with regard to rotational angle will equal:

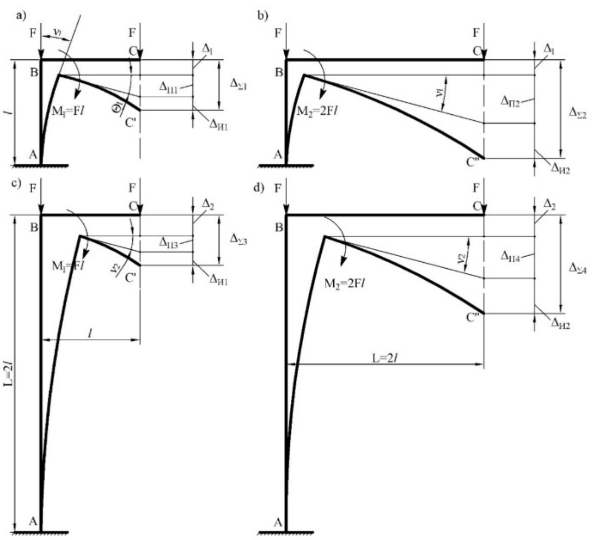

*Fig. 2. Curved rods (frames) being compared with different part length ratios AB:BC: a – 1:1; b – 1:2; c – 2:1; d – 2:2.*

$$K^{П}_{Жi} = \frac{\Delta_{И1}}{\Delta_{Пi}}, \qquad (7)$$

Taking into account the particular criteria (1), (2), (7), we get the total stiffness ratio for any current position of the manipulated object

$$K_{Жi} = K^{P}_{Жi} + K^{П}_{Жi} + K^{И}_{Жi}. \quad (8)$$

Once components are substituted in the formula (3), versions comparison results (Fig.2) according to the stiffness ratios will be as follows—see the Table 1.

The stiffness ratios of curved rods (frames) compared, according to Figure 2

Table 1

| Version No. | $K^{P}_{Жi}$ | $K^{П}_{Жi}$ | $K^{И}_{Жi}$ | Total $K_{Жi}$ |
|---|---|---|---|---|
| 1 | 1 | $\frac{2}{3}$ | 1 | $2\frac{2}{3} = 2{,}33$ |
| 2 | 1 | $\frac{1}{3}$ | $\frac{1}{8}$ | $1\frac{11}{24} = 1{,}58$ |
| 3 | $\frac{1}{2}$ | $\frac{1}{6}$ | 1 | $1\frac{2}{3} = 1{,}67$ |
| 4 | $\frac{1}{2}$ | $\frac{1}{12}$ | $\frac{1}{8}$ | $\frac{17}{24} = 0.71$ |

The location of load application spot changes in different positions of the manipulated object, so stiffness of the system leading to a change in the end link deformation (which is advisable to evaluate by the stiffness instability factor (criterion)) will be:

$$K_{\delta Жi} = \frac{2(\Delta_{\Sigma i\max} - \Delta_{\Sigma i\min})}{\Delta_{\Sigma i\max} + \Delta_{\Sigma i\min}}, \quad (9)$$

where $\Delta_{\sum i\max}$ и $\Delta_{\sum i\min}$ are the maximum and the minimum deformation values respectively at the spot in the area of object manipulation, to which the load F is applied.

With a total stiffness index (8) and instability factor calculated, we can formulate an integral stiffness factor:

$$K^{*}_{Жi} = \frac{K_{Жi}}{1 + K_{\delta Жi}}. \quad (10)$$

In this case, when selecting the best wire-frame arrangement, we may deal with a multi-criterial problem of structural circuit synthesis though [2,4,11].

### 3. Solution examples

Let's consider two examples. Example1. Determine the index $K_{\delta Ж2}$ for a floor-mounted industrial robot (Fig. 3) complying with two design models for maximum $\Delta_{\sum 2\max}$ (fig.2, b) and minimum $\Delta_{\sum 1\min}$ at the given deformation distance H. (fig. 2,a) (table 1):

$$\Delta_{\Sigma 2\max} = \frac{Fl}{ES} + \frac{Fl^3}{EI} + \frac{8Fl^3}{3EI}; \quad (11)$$

$$\Delta_{\Sigma 2\min} = \frac{Fl}{ES} + \frac{Fl^3}{2EI} + \frac{Fl^3}{3EI}. \quad (12)$$

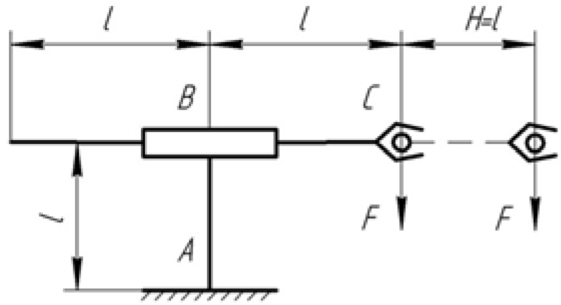


***Fig. 3.*** *The diagram of a floor-mounted industrial robot*

Substituting (11) and (12) into (9) and rearranging the equation, we get

$$K_{\delta Ж2} = \frac{5{,}66}{0{,}125\frac{d^2}{l^2} + 4{,}5}, \quad (13)$$

For $\frac{d}{l} = B$:

$$K_{\delta Ж2} = \frac{5{,}66}{0{,}125B^2 + 4{,}5}. \quad (14)$$

Example 2. Determine the index $K_{\delta Ж}$ for a floor-mounted industrial robot with flat PSM of biglide type (Fig. 4).

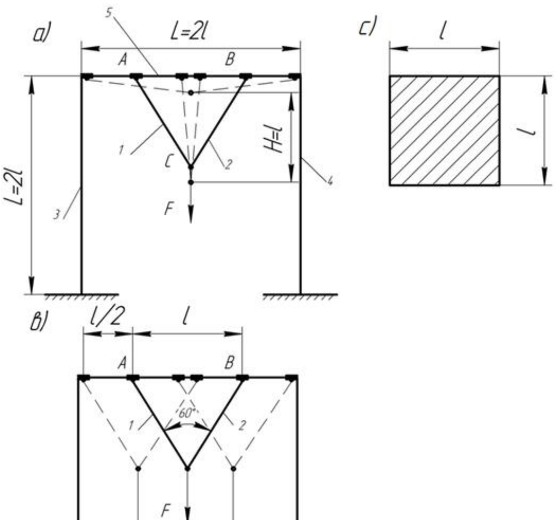


***Fig. 4.*** *The diagram of floor-mounted industrial robot with flat PSM of biglide type and the manipulated object moving in vertical (a), horizontal (b) directions, forming a square working area (c).*

Let's accept that that the maximum deformations (elastic displacement of the point C) of the manipulated object will be under the link rods 1 and 2 brought together (Figure 4a and 5a):

$$\Delta_{\Sigma\max} = \Delta_{H1\max} + \Delta_{2\max} = \Delta_1 + \Delta_{H1\max} + \Delta_{2\max}, \quad (15)$$

where $\Delta_{H1\max} = \Delta_1 + \Delta_{H1\max}$ is the maximum deformation of the carrier system in the point of application of force F; $\Delta_1$ is compression deformation of the vertical struts 3 and 4; $\Delta_{H1\max}$ is the maximum bending deformation of the portal frame—cross head 5; $\Delta_{2\max}$ is the maximum stretching strain of the link rods 1 and 2 of the (biglide's) fixed length.

In accordance with [6, 18] let's define the components $\Delta_1 = \frac{Fl}{ES}; \Delta_{H1\max} = \frac{Fl^3}{6EI}; \Delta_{2\max} = \frac{Fl}{2ES}$, after substitution and rearrangement the equation, we get:

$$\Delta_{\Sigma\max} = \frac{7{,}5Fl}{Ed^2} + \frac{3{,}3Fl^3}{Ed^4}. \quad (16)$$

Let's accept that the minimum elastic displacement in the point C will be under the link rods 1 and 2 (Fig. 4, b and 5, b, c) set apart in one of the extreme positions on the cross head 5:

$$\Delta_{\Sigma\min} = \Delta_{H1\min} + \Delta_{2\min} = \Delta_1 + \Delta_{H1\min} + \Delta_{2\min}, (17)$$

where $\Delta_{H1\min} = \Delta_1 + \Delta_{H1\min}$ is the minimum deformation of the carrier system in the spot of application of the force F; $\Delta_{H2\min}$ is the minimum bending deformation of the cross head 5; $\Delta_{2\min}$ is the minimum elastic displacement in the point C of the set apart link rods 1 and 2 jointing.

In accordance with [6, 18] let's define missing components $\Delta_{2H\min} = \frac{0{,}19Fl^3}{EI}; \Delta_{2\min} = \frac{2}{3}\frac{FLl}{ES}$, substituting to (17), and after rearranging the equation we get:

$$\Delta_{\Sigma\min} = \frac{2{,}09Fl}{Ed^2} + \frac{0{,}38Fl^3}{Ed^4}, \quad (18)$$

After substituting (16) and (18) to (9) and rearranging the equation, we get:

$$K_{\delta Ж} = \frac{10{,}82B^2 + 5{,}84}{9{,}59B^2 + 3{,}6}, \quad (19)$$

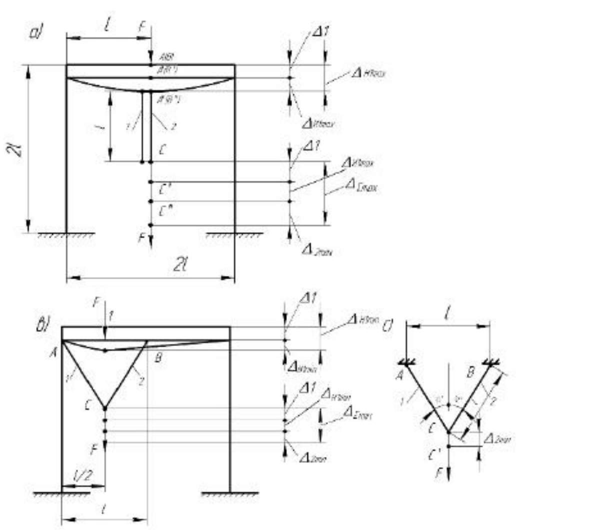

*__Fig. 5.__ Calculation models for determination of instability factor for a gantry robot with a biglide PSM (fig. 4) in positions of the maximum (a) and the minimum (b, c) manipulated object elastic displacement in the point C of the force F application area.*

For these examples, all the indices (criteria) defining the stiffness at the stage of structural circuit synthesis can be easily identified by the methods suggested. Still, complication of the task with transition to reviewing spatial layout of a multi-criterial problems [2, 4, 5] will require the tasks simulation with the help of computers and, in particular, PCs. At a later stage, it is advisable to consider formalized description of other indices (criteria) selected from the previously mentioned functional, technological, economic and ergonomic ones.

## CONCLUSIONS

In this work, the integrated stiffness index is suggested, with a round rod of the same cross-section (diameter) and longitudinal (length) dimensions, and an assigned ratio as a dimensionless quantity, used as rating index. This enables selection of the best solutions on the stage of structural circuit synthesis for wire-frame arrangements of industrial robots with and without mechanisms of parallel structure.

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