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TIME AND FREQUENCY ANALYZES OF A FREQUENCY LOCKED LOOP

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ABSTRACT

Unlike the Phase Locked Loops (PLL) based on the Time Recursive Processing, this paper describes one model of Frequency Locked Loop (FLL), which is based on the Time Nonrecursive Processing of the input periods. FLL represents linear discrete system, which is described by two difference equations. All analyzes in time domain are performed using Z transform approach. The analyzes in frequency domain are performed by matlab tools which are dedicated to design and application of digital filters. It was shown that FLL is very powerful in the tracking and predicting applications. In this paper, special attention was devoted to finding ways to use the powerful matlab tools in the analysis of FLL. The simulation of FLL functioning proofed the correctness of the mathematical analyzes. The realization of FLL was described. The oscilloscope picture, made on the realized model, is presented.

INTRODUCTION

Unlike the described algorithms in refs. [1-9], this approach of FLL is an open loop system. From the aspect of the algorithm used, based on difference equations, it functions similar as FIR digital filter. Although digital filters process amplitude samples of the input signal, and FLL use periods of an input signal, it is very useful to understand the similarities and differences between these two physically different systems in order to utilize, as much as possible, power digital filter theory and matlab software tools in further development and application of FLL. FLL, described in this paper, calculate and generates an output period using the measurement and the processing of the input periods only. The term "nonrecursive" is borrowed from the theory of digital filters. Actually, finite impulse response (FIR) digital filters calculate a next output only using the input samples, without to take in account the previous filter outputs. Such processing was called "nonrecursive". Infinite impulse response (IIR) digital filters use in calculations both, the input samples and the previous outputs as well. This kind of the processing was called "recursive". All refs. [1-9], describing different applications, use recursive processing of the input and output periods.

The theory and techniques for the developing of FLL are basically very similar to the demonstrated one through refs. [1-9]. The applicability of this approach is very wide. Frequency multiplier is described in [1]. Time shifters are described in [2, 3] and time/phase shifting in [4]. PLL and FLL for noise rejection are described in [5-7]. A wide range of tracking and prediction applications is described in [5, 6, 8]. Most of the algorithms described in [1-9] are suitable for usage in a software form. Such a software predictor is described in [9]. The articles and books [10-15] are used as theoretical base, for electronics implementation and for the development necessities.

DESCRIPTION OF FLL

General case of an input signal Sin and an output signal Sop of FLL is shown in Fig. 1. The time difference τ_k is used in Fig. 1 instead of the phase difference. The periods TI_k and TO_k, as well as the time difference τ_k, occur at discrete times t_0, t_1, …t_k, t_{k+1}, which are defined by the falling edges of the pulses of Sop in Fig. 1. The main difference equation describing the functioning of FLL is presented

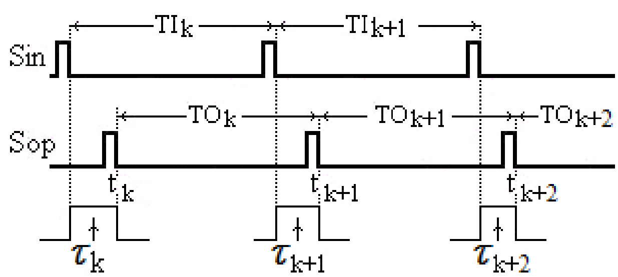

Fig. 1. Time relation between variables

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by eq. (1), where "a" and "b" are the system parameters. The natural relations between variables, which come out from Fig. 1, are given by eqs. (2) and (3). Note that eq. (3) is just eq. (2), shifted for one step. Eq. (3) will be

$$TO_{k+2} = a \cdot TI_k + b \cdot TI_{k+1} \tag{1}$$

$$\tau_{k+1} = \tau_k + TO_k - TI_k \tag{2}$$

$$\tau_{k+2} = \tau_{k+1} + TO_{k+1} - TI_{k+1} \tag{3}$$

taken in account for the simulation of FLL. According to the equations (1) and (2), FLL has two output discrete variables, which describe the behaviour of FLL in terms of TI_k. The output variables are $TO(k+1)$ = f[$TI(k)$] and $\tau(k+1)$ = f[TI(k)]. To analyze the conditions under which the described system possesses the properties of FLL, the Z transforms of eqs. (1) and (2) are presented in eqs. (4) and (5) respectively, where TO_1, TO_0 and τ_0 are the initial values of TO_k and τ_k. Note that, according to eq. (1), $TO_1=bTI_0$. Substituting $TO_1=bTI_0$ into eq. (4), $TO(z)$ was found and presented in eq. (6). Substituting $TO(z)$ from eq. (6) into eq. (5), $\tau(z)$ was calculated and presented in eq. (7). Note that, in order to shorter the mathematical procedure, eq. (7) is reached taking in account that the relation between the system parameters must be a+b=1, shown later on, in eq. (12). Two transfer functions, which describe FLL, can be now recognized. The first one is H_{TO}(z), shown in eq. (8), which describes the behaviour of the output period in terms of the input period. The second one H_τ(z), shown in eq. (9), describes the behaviour of the time difference in terms of the input period.

$$z^2TO(z) - zTO_1 - z^2TO_0 = aTI(z) + zbTI(z) - zbTI_0 \tag{4}$$

$$z\tau(z) - z\tau_0 = \tau(z) + TO(z) - TI(z) \tag{5}$$

$$TO(z) = TI(z)\frac{(a+zb)}{z^2} + TO_0 \tag{6}$$

$$\tau(z) = -TI(z)\frac{z-(b-1)}{z^2} + \frac{TO_0 + z\tau_0}{z-1} \tag{7}$$

$$H_{TO}(z) = \frac{TO(z)}{TI(z)} = \frac{(a+zb)}{z^2} \tag{8}$$

$$H_\tau(z) = \frac{\tau(z)}{TI(z)} = -\frac{z-(b-1)}{z^2} \tag{9}$$

ANALYSES OF FLL

It is necessary now to investigate the conditions under which the described system possess the properties of FLL. Let us remember that a FLL generates the output pulse rate whose frequency tends to reach the frequency of the input pulse rate during the transient state. FLL is in the stable state when the output frequency becomes either equal or in certain pre-defined relation to the input frequency. FLL does not care about the phase difference between the input and the output signals. FLL regulates only the frequency of the output signal. The phase difference of FLL depends on the initial conditions and FLL parameters. Unlike FLL, PLL regulates both the frequency and the phase difference between the input and the output signals, at the same time. In most of the applications, classical PLL tends to equalize both frequency and phase of the input and the output signals. However, taking in account results in refs. [2, 3], the phase difference of Time Recursive PLL between the input and the output signals can be regulated by the system parameters. This phase difference can take any value, but it does not depend on the initial conditions of the variables. Generally, some of Time Recursive PLL can control the phase difference between the input and output signals by the system parameters. Note that thereby, the phase difference does not depend on the initial conditions. Otherwise, the system would represent FLL.

The step analyzes is the most suitable approach for the investigation of the properties of the system described. Let us suppose that the step input is TI(k) = TI = constant. Substituting the Z transform of $TI(k)$ i.e. $TI(z) = TI\cdot z/(z-1)$ into eq. (6) and using the final value theorem, it is possible to find the final value of the output period $TO_\infty = \lim TO(k)$ if $k\to\infty$, using $TO(z)$:

$$TO_\infty = \lim[(z-1)TO(z)]_{z\to 1} = TI(a+b) \tag{10}$$

Substituting now $TI(z) = TI\cdot z/(z-1)$ into eq. (7) and using the final value theorem, it is possible to find the final value of the time difference $\tau_\infty = \lim \tau(k)$ if $k\to\infty$, using $\tau(z)$:

$$\tau_\infty = \lim[(z-1)\tau(z)]_{z\to 1} = TI(b-2) + TO_0 + \tau_0 \tag{11}$$

It can be concluded, according to eq. (10), that the described system can possess the property of a FLL, if the system parameters satisfy eq. (12). Note that if eq. (12) is satisfied, $TO_\infty=TI$, i.e. for the stable FLL, the output frequency is equal to the input frequency. Equation (11) confirms that the system possesses the properties of FLL, since τ_∞ depends on the initial conditions. It comes out that the system does not possess the property of a PLL.

$$a + b = 1 \tag{12}$$

It is of interest to analyze now, whether FLL is able to track the ramp input. To estimate this, it is necessary to determine well known velocity error K_V, providing that the input period is the ramp function $TI(k)=TI_V(k)=c\cdot k$, where "c" is a time constant. Note that $TI(z)=TI_V(z)=Z(c\cdot k)=cz/(z-1)^2$. It is known, that velocity error $K_V = \lim[TO_V(k)-TI_V(k)]$ for $k\to\infty$. One more suitable expression for velocity error is $K_V = \lim TI_V(k)[H_{TO}(k)-1]$ for $k\to\infty$. Using the condition a+b=1, the final value theorem and $H_{TO}(z)$ given by eq. (8), K_V is calculated and shown in eq. (13). According to eq. (13), FLL is able to track the velocity input with the constant error. However, if b=2 (a=1-b=-1), $K_V=0$, i.e. FLL tracks the velocity input without any error.

$$K_V = \lim\{(z-1)TI_V(z)[H_{TO}(z)-1]\}_{z\to 1} = c(b-2) \tag{13}$$

Let us now determine the behaviour of $\tau_V(k)$ for the velocity input, if $k\to\infty$. Taking in account b=2 and a=-1, and using the final value theorem, $\tau_{V\infty}=\lim \tau_V(k)_{k\to\infty}$ is calculated using $\tau_V(z)$ and shown in eq. (14). The expression $\tau_V(z)$ is found out by the substitution of $TI(z)=TI_V(z)=cz/(z-1)^2$ in eq. (7). According to eq. (14), $\tau_{V\infty}$ is the constant. Besides the initial conditions TO_0 and τ_0, $\tau_{V\infty}$ depends on the time constant "c", which is the slope of the ramp input function.

$$\tau_{V\infty} = \lim[(z-1)\tau_V(z)]_{z\to1} = -c + TO_0 + \tau_0 \quad (14)$$

It is worth checking whether FLL is able to track the acceleration input function $TI(k)=TI_A(k)=c\cdot k^2$. Note that, in this case, $TI(z)=TI_A(z)=Z(c\cdot k^2) = cz(z+1)/(z-1)^3$. It is necessary to calculate now the acceleration error $K_A=\lim[TO_A(k)-TI_A(k)]$, for $k\to\infty$. One more suitable expression for velocity error is $K_A = \lim TI_A(k)[H_{TO}(k)-1]$ for $k\to\infty$. Taking in account the values of parameters b=2 and a=-1, than using the final value theorem and $H_{TO}(z)$ given by eq. (8), K_A is calculated and shown in eq. (15). According to eq. (15), FLL is able to track the acceleration input, but with the constant time error $K_A = -2c$.

$$K_A = \lim\{(z-1)TI_A(z)[H_{TO}(z)-1]\}_{z\to1} = -2c \quad (15)$$

The results of the step analyses will be supported by the simulation of FLL operations. This simulation are to prove the mathematical results and to enable better insight into the procedure and the physical meaning of the variables described. All discrete values in simulations were merged to form continuous curves. Note that all variables in the following diagrams were presented in time units. The time unit can be, μsec, msec or any other, but assuming the same time units for *TI*, *TO*, τ and „c“, it was more suitable to use just “time unit” or abbreviated “t.u” in the text. It was more convenient to omit the indication „t.u“ in diagrams. All simulations were performed using eqs. (1) , (2) and (3). The simulations of *TO(k)* and *τ(k)* for the step input TI_k=10 t.u, are shown in Figure 4a. All values for three cases of different parameters „a“ and „b“, initial conditions and final values are shown in Figure 4a. The system parameters satisfy eq. (12) in all cases and consequently, the output periods reached the input periods. According to eq. (11), using the values of parameters and the initial conditions presented in Fig. 4a, it can be calculated $\tau_{1\infty}$ = TI(b-2)+TO_0+τ_0 = 10(0.1-2)+3+0 = -16 t.u. This result agrees with the simulated $\tau_{1\infty}$, shown in Fig. 4a. In the same way, it can be calculated that $\tau_{2\infty}$ = -4 t.u, and $\tau_{3\infty}$ = 6 t.u. Note that the calculated values $\tau_{2\infty}$, and $\tau_{3\infty}$ also agree with the simulated $\tau_{2\infty}$, and $\tau_{3\infty}$ presented in Fig. 4a.

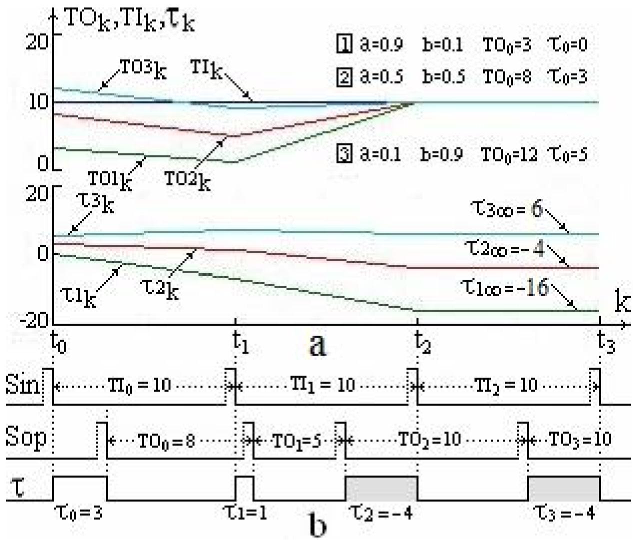

Fig. 4 a. Transition states of FLL for the step input and different system parameters, b. Real time presentation of Sin, Sop and τk for the simulated case Nr 2.

These simulation results prove the correctness of the mathematical description and step analyses. The real time relation between Sin, Sop and τ_k, for the simulated case Nr. 2, is shown in Figure 4b. For the stable FLL, period TO_∞ = TI = 10 t.u and τ_∞ = -4 t.u. Note that FLL is very fast. It takes only two steps to reach the stable state.

ANALYSIS IN FREQUENCY DOMAIN

For the analysis of FLL in frequency domain, matlab commands, devoted to digital filter design, are used. Since FLL is described by two transfer functions $H_{TO}(z)$ and $H_\tau(z)$, shown in eqs. 8 and 9, using matlab command "freqz", frequency responses in the regain (0, pi) [rad], from both transfer functions, are generated for the parameters a=-1 and b=2 and presented in Figs. 5a and 5b respectively. The frequency responses consist of magnitude and phase responses. The sampling frequency f_s=200 Hz corresponds to the whole region (0, 2pi) [rad], so that fs/2=100 Hz, covers the region (0, pi) [rad] in Fig. 5.

For the comments about Fig. 5, time presentations of TI and TO as well as the spectrums of TI, TO and τ, shown in Fig. 6, will be used. The input signal is the input period TI, presented in Fig. 6a, as TI(k+1)=10+6·sin[(2pi/f_s)·f_m·k)]. In fact, this is the constant period of 10 t.u., which is modulated by samples of sinus signal, whose amplitude is 6 t.u. and frequency fm = 10 Hz. Number of time steps is chosen to be k=200=fs. The angular sampling step is ws = 2pi/200 [rad]. Since 200 frequency sampled steps covers region of one period, (0, 2pi) [rad], it means that every of 10 periods of sinus signal will be sampled by 200 samples/10 periods=20 samples/period. This provides sufficiently good resolution of TI in Fig. 6a for this analysis. FLL generates the output TO, which is calculated according to eqs. 1, 2 and 3. Note that TO exactly tracks TI with delay of one step, Fig. 6a. Matlab commands "fft" and "stem" are used for the generation of the spectrums of TI, TO and τ in Fig. 6b.

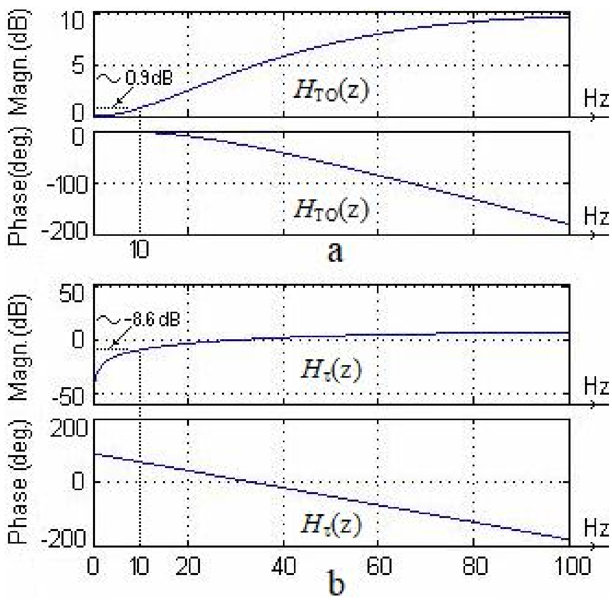

Fig. 5 a. Frequency responses - HTO(z)
b. Frequency responses - Hτ(z)

These spectrums present the absolute values of amplitudes, covering the whole region (0, 2pi) [rad]. They appear as positive values in the symmetric second half (pi, 2pi) [rad]. The constant of 10 t.u., as a part of the input signal *TI*(k+1), corresponds to zero frequency component. This constant appears as very strong amplitude with the frequency of 0 Hz in spectrums of *TI* and *TO*. Besides the constant of 10 t.u., *TI*(k+1) consist of the sinusoidal signal, whose frequency amplitude can be seen in spectrums of *TI*, *TO* and τ at 10 Hz. Time amplitude of TO spectrum at frequency of 0 Hz is practically the same as in spectrum of TI, because FLL attenuation, shown in Fig. 5a for H_{TO}, is 0 [dB] at 0 Hz.

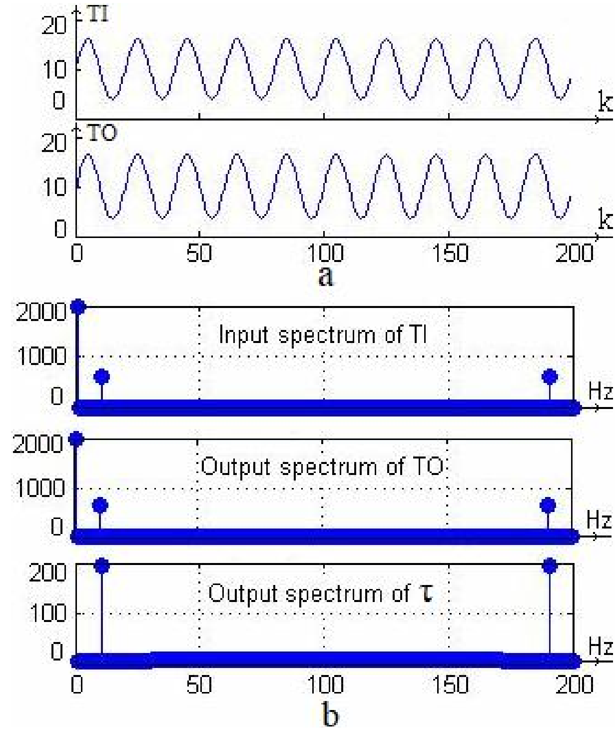

Fig. 6 a. Time presentation of TI and TO, b. Spectrums of TI, TO and τ

However the time amplitude at frequency of 10 Hz is slightly amplified at the output TO, because the magnitude, at Fig. 5a for HTO, is about 0.9 [dB]. According to Fig. 6b, the amplitude of the input sinus signal at 10 Hz is about 560 t.u. and in the spectrum of τ, this component amounts about 200 t.u. If we express this attenuation in [dB], it gives 20 log(200/560) ~ -8.6 [dB]. This corresponds to the attenuation of -8.6 [dB], shown in Fig. 5b. Unlike very strong amplitude at the frequency of 0 Hz in the spectrum of TO, this component completely disappeared in the spectrum of τ in Fig. 6b, because FLL attenuation, shown in Fig. 5b for Hτ(z), is about 50 [dB] at 0 Hz. This fact means that the width of time difference τ contains the input sinus signal, but without zero frequency component. However, according to eq. (11), τ depends on the initial conditions too. But the difference between two adjacent τ, i.e. τk+1-τk, will eliminate the initial conditions, since their influence is the same in every τ, if FLL is in the stable state. It comes out that FLL can be used as a demodulator of the sinusoidal signal.

REALIZATION OF FLL

According to the previous analyzes, FLL possesses the powerful tracking performances for a=-1 and b=2. If we substitute these values of the parameters into eq. (1), it will be transformed into eq. (16). If we now multiply, at the same time, all of its members by clock frequency fc, eq. (16) will be transformed into eq. (17). The functional scheme of FLL, which comes out from eq. (17), is presented in Fig. 7. According to eq. (17), the input period TIk is measured by clock with frequency fc, the input period TIk+1 is measured by clock with frequency 2fc and the output period TOk+2 is generated by clock with frequency fc. FLL consists of Recursive Calculation Model (RCM) and Programmable Period Generator (PPG). RCM calculates Nb in binary form, according to the right side of eq. (17), and PPG generates the output period in the next step. PPG is described in refs. [2-4]. For the realization of RCM, presented in Fig. 2, the same technique was used as in refs. [1-9].

$$TO_{k+2} = -TI_k + 2TI_{k+1} \qquad (16)$$

$$f_c \cdot TO_{k+2} = -f_c \cdot TI_k + 2f_c \cdot TI_{k+1} \qquad (17)$$

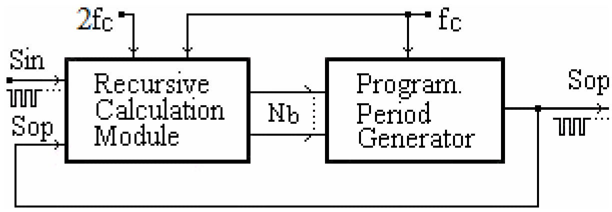

Fig. 7. The functional scheme of FLL.

The real time functioning of FLL is presented in Fig. 8. The oscilloscope picture is made on the realized eight-bit FLL. The voltage waveforms in Fig. 8 are taken when FLL was in the stable state. For this purpose step input was chosen *TI*=0.1 ms (f_{in}=1/*TI*=10 kHz), a=-1 and b=2. Clock frequency corresponding to parameter "a" was f_c = 110 kHz. Clock frequency corresponding to parameter "b" was $2f_c$ = 220 kHz. The ratio $TI/t_c = f_c/f_{in}$ = 110 kHz/10 kHz = 11. This ratio can be noticed in Fig. 8.

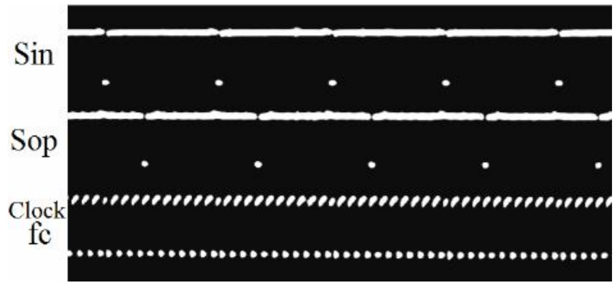

Fig. 8. The loop is in the stable state.

CONCLUSION

This paper is closely related to the recently published articles in ref. [1-9]. Due to the fact, that this FLL is based on the measurement and processing of the input periods only, it is simpler for the realization in comparison to those described in ref. [1-9]. At the same time, it takes FLL only two steps to reach the stable state for any kind of input. It was shown that this FLL can be very efficiently used for the tracking of the step, the ramp and the acceleration functions. It is especially suitable for those applications, which require fast FLL with very short transient time. This FLL is scalable to the very strict requirements in the fields of tracking and predicting.

Although the FLL and the digital filter represent different types of systems, since the first is based on time processing and the other one is based on the processing of amplitudes, the article showed that matlab tools, devoted to the design of FIR digital filters, can be used to analyze the FLL in the frequency domain. All it takes is to understand the physical aspects of the whole process and to identify the meanings of FLL variables in matlab tools. Using matlab tools, wide options for new analyzes and new applications of FLL are provided. Such one is described in this article. It was discovered that FLL can be used as a demodulator.

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